Multifractal analysis of complex boundaries

Spring 2019

[•] Context. Multifractal analysis is a mathematical modeling and analysis framework that describes a function \( f : \mathbb{R}^d \rightarrow \mathbb{R} \) based on the global geometry of the fluctuations of their pointwise regularity \([1, 2]\). In practice, after averaging, this assumes the absence of characteristic scales and intermittent, complex transient dependence. Essentially, multifractal analysis relies on the identification of the power law exponents \( \zeta(q) \) that relate the \( q \)th moments of multiresolution coefficients \( T_j(a, x) \), such as discrete wavelet coefficients, at different scales \( a \),

\[
\sum_k |T_X(a, x = k)^q| \simeq c_{\phi[a]}^{\zeta(q)}. \quad (1)
\]

Multifractal analysis is an active area of research and is becoming a standard data analysis tool that naturally encompasses rich nonlinear and multiscale data properties. It has been successfully applied for the analysis of real-world signals and images in a broad range of contexts, from Physics (e.g. material science \([3]\)) over man-made signals (Internet traffic \([4]\), perception and Art \([5]\)) to biomedicine \([6]\) to quote but a few examples (see also \([2]\) and references therein).

[•] Limitations. Despite such massive successes in applications, today’s state-of-the-art for practical multifractal analysis of images remains essentially restricted to the study of a single homogeneous texture. Yet, in many applications, the characterization of the boundaries separating different textures in an image is of central interest. To give a concrete example, the complex and heterogeneous interaction of tumours with their neighboring microenvironment is believed to play an important role in the development of resistance to treatment therapy, and the quantitative analysis of such boundaries could reveal crucial information.

[•] Objectives. Building on preliminary works, the goal of the internship is to explore models and tools for the multifractal analysis of complex boundaries:

1. A formalism for studying multifractal boundaries has been theoretically proposed in \([7, 8, 9]\). It relies on specific multiresolution coefficients, the \( p \)-leaders, that have already been studied for the multifractal analysis of image texture \([10, 11]\). Making use of existing MATLAB implementations, the first objective of this internship will be to study the use of these tools for the analysis of simple boundary models.

2. The second objective consists in studying synthetic models for multifractal boundaries. Both deterministic models (such as generalizations of the famous fractal Von Koch curve) or stochastic models (for instance, the level sets of complex textures) will be explored theoretically and validated numerically.

[•] Perspectives. This internship is part of larger national research programme aiming at the multifractal modeling and analysis of biomedical images. The developed models, tools and concepts could therefore also be put to test on real-world biomedical images. Depending on the outcome of the internship, it is possible that the topic will be pursued by a PhD project (funding secured).

[•] Requirements. Candidates should have a solid background in mathematics and statistics, and should be operational with MATLAB. An interest in the analysis and modeling of (real-world) images is of advantage but not indispensable.

[•] Contact. This internship will be co-advised by:
- Herwig Wendt, Researcher, CNRS, Institut de Recherche en Informatique de Toulouse.
- Clothilde Melot, Maître de Conférence, Institut de Mathématiques de Marseille, Université Aix Marseille.

[•] Application. All applications must be sent electronically to the advisors (minimum: motivation letter, CV).
References


